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COUNTERPARTY VALUATION ADJUSTMENTS

Harvey Stein, MD, Counterparty and Credit Risk
Kin Pong Lee, Quantitative Finance, Interest Rate Derivatives
Bloomberg LP

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Contents

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The financial crisis that began in 2007 has highlighted the importance of assessing counterparty credit risk. Regulations, accounting practices and investment practices are all being reshaped to better manage counterparty risk.

Here we review the need for counterparty credit risk analysis, focusing on accurate computation of the counterparty valuation adjustment (CVA). We:

- provide a general framework for computing CVA,
- relate the CVA to the value of a portfolio of options,
- compare the CVA of bonds to swaps,
- analyze other approaches to CVA management (such as discount shifts, current exposure and bilateral CVA), and
- discuss hedging methodologies.

Introduction

Despite the recent market upheavals, the OTC derivatives markets continue to comprise one of the largest components of the financial markets, with an overall outstanding notional of $547 trillion in December 2008, 70% of which are in interest rate derivatives. As of June 2009, this grew to $605 trillion. And in spite of market contractions, gross values in the OTC markets are up. From June 2008 to December 2008, OTC gross market value increased 60%, from $20 trillion to $32 trillion (Bank for International Settlements, June 2009). Interest rate derivatives' gross market value doubled from $9 trillion to $18 trillion.

Prompted by the desire to weather or even reduce market turmoil, regulations, accounting practices and investment practices have been under reevaluation. In particular, approaches for analyzing and mitigating counterparty risk have garnered renewed interest. Regulators have been advocating greater usage of clearing houses. Accounting boards have been refining and codifying fair market valuation, placing additional emphasis on careful consideration of counterparty risk. The International Accounting Standards Board (IASB) has even issued a request for comment on counterparty risk calculation methodologies. And investors and traders have been trying to better factor some notion of counterparty risk into their trading and risk management practices.

Here we will investigate the notion of counterparty risk and the associated counterparty valuation adjustment (CVA) in the fixed income markets. We will outline the CVA calculation, detail the underlying model assumptions, give examples of the calculation, and discuss the impact the CVA has in the value of these instruments.

Counterparty Risk

Counterparty risk is the exposure the investor has to loss due to a specific counterparty failing to meet his contractual obligations. This is the default risk of a position or portfolio.

When holding an uncovered contract with a counterparty, one's immediate counterparty exposure is to the loss (if any) that would incur if the counterparty were to default. If the investor is long a bond, then the exposure is to the bond issuer, and the loss that would incur

is the value of the bond minus the amount that would be recovered by the investor. If the investor is short the bond, then he bears no counterparty risk.

The immediate exposure on an uncovered swap is different. With a swap, one is simultaneously long the receiving leg and short the paying leg. At some times, this one contract can be an asset and thus exposed to counterparty default, while at other times it can be a liability and engender no counterparty exposure at all.

Another difference between bond exposure and swap exposure is in recovery. With bonds, investors receive a percentage of the principal (recovery on principal) (Altman, et al., Nov/Dec 1996). The remaining interest payments of the contract are lost. With a swap, as per the ISDA master agreement, recovery is on the market value of the swap, which values both principal and interest payments (Cooper, et al., June 1991).

The final difference between bond and swap counterparty exposure is that swap counterparty exposure is often modified or mitigated by other contracts. If the parties involved have activated the netting agreement in the ISDA master agreement, then the overall exposure at a given time is not to the loss on each individual contract, but to the net value of all contracts covered by the netting agreement. In particular, without a netting agreement, one would receive recovery on each swap with positive value, while still owing the full market value of swaps with negative values. With a netting agreement in place, swaps with negative value will decrease the overall exposure (Brigo, et al., 2005)

The other agreement that modifies credit risk is the ISDA credit support annex (CSA). When undertaken, this agreement stipulates that positions must be collateralized and details exactly how this must be done. The parties maintain an account with collateral against the value of the swap (or net value of all the instruments under the netting agreement). When the difference between the net value of the securities and the amount of collateral posted exceeds the margin requirements, additional collateral must be posted to make up the difference. This limits exposure to the size of market moves before additional posting is demanded, plus the size of the margin requirement (Alavian, et al., March, 2009).

Counterparty Valuation Adjustment – CVA

Exposure to default is quantified by the counterparty valuation adjustment (CVA) (Alavian, et al., March, 2009). This is the price deficit of the instrument that arises from default risk. It is the difference between the price of the instrument and the price it would have were the counterparty free of default risk (the risk free price minus the risky price). This is the cost of hedging the default risk, or equivalently, the price of the embedded default risk.

Counterparty risk calculations are fairly straightforward for instruments and portfolios that are long-only. In a long only position (like a bond position), counterparty risk can be judged by using models that are able to incorporate a discount curve shift. CVAs depend on interest rates and default risks, and are only sensitive to volatility if the underlying structure itself is volatility sensitive.

For positions that can be long or short depending on market conditions, counterparty risk calculations are more complicated. This applies to interest rate swaps. For these instruments, CVAs do not just depend on interest rates and default risk. They also depend on volatility.

For example, consider a five-year at-the-money receive fixed interest rate swap in a flat interest rate environment. At zero volatility, the swap will for the most part have a nearly zero market value, and thus little default risk. However, at high volatility, the swap has the potential to be fairly valuable in the future. Under those conditions, default would cause a substantial loss.

Swaps, in that exposure is to the market value and not to the principal, also have more exposure than bonds to the shape of the yield curve. Even at zero volatility, if the swap curve is steep, then the swap can be heavily in-the-money for much of its life, and thus will exhibit far more counterparty risk than in a flat curve environment.

Modeling the CVA

Consider a contract with a given counterparty[1] that matures at time T and has price process $V(t)$ when neglecting the embedded default risk. Let $P(t)$ be the "principal" at time t. For example, if this is a position in a bond issued by a risky company, then $V(t)$ is the price at time t of the equivalent risk-free bond and $P(t)$ is constant if the bond is not amortizing. Let R_V (R_P) be the recovery rate on the price (principal), which we assume to be a known constant.

If the counterparty were to default right now and the position currently has positive value ($V(0) > 0$), then the investor receives $R_v \times V(0) + R_P \times P(0)$ for the contract, effectively losing $(1 - R_V)V(0) - R_P P(0)$. If the value is negative, there is no loss – the investor still owes the liability. Thus, the immediate exposure to default is:

$$(1 - R_V)\max(V(0), 0) - R_P P(0) 1_{V(0)>0},$$

where $1_{V(0)>0}$ is the indicator function (it is 1 when $V(0) > 0$ and otherwise 0). Similarly, if τ is the time of default, then our loss at that time is:

$$(1 - R_V)\max(V(\tau), 0) - R_P P(\tau) 1_{V(\tau)>0}$$

The cost of this payoff is the counterparty valuation adjustment (CVA) for the contract. It is the cost of the default risk, or equivalently, the cost of hedging the counterparty risk. The first term is the loss after recovery on the market value (CVA_V). The second term is the loss after recovery on the principal (CVA_P). For individual securities, either R_V or R_P will typically be zero. For a portfolio with mixed recoveries, one would need to sum over the value and principal processes of the constituents.

[1] This could also be a portfolio of securities, all under a netting agreement with a given counterparty.

In the case of being long bonds, the market value recovery is zero ($R_V = 0$), and the market value is nonnegative, so the loss at default time reduces to:

$$V(\tau) - R_P P(\tau).$$

In the case of a swap, where recovery is on the market value alone ($R_P = 0$), the loss at default time reduces to:

$$(1 - R_V)\max(V(\tau), 0)$$

The quantity $\max(V(t), 0)$ is the payoff of a call option to purchase the contract at time t. Taking expectations with respect to the risk neutral measure for a numeraire N, we see that this component of the CVA, namely the loss from recovery on the market value, is given by:

$$N(0)E\left[(1 - R_V)\frac{\max(V(\tau),0)}{N(\tau)}1_{\tau<T}\right]$$

$$= (1 - R_V)\, N(0)E\left[\int_0^T \frac{\max(V(t), 0)}{N(t)}\delta(t-\tau)dt\right]$$

$$= (1 - R_V)\, N(0)\left[\int_0^T E\left[\frac{\max(V(t), 0)}{N(t)}\delta(t-\tau)\right]dt\right],$$

where δ is the Dirac delta function. If under the equivalent martingale measure, default is independent of both the contract value and the numeraire, then the expectation of the above product is the product of the expectations, and we see that the CVA_V is:

$$(1 - R_V)\, N(0)\left[\int_0^T E\left[\frac{\max(V(t), 0)}{N(t)}\right] E[\delta(t-\tau)]\, dt\right]$$

$$= (1 - R_V)\left[\int_0^T N(0)E\left[\frac{\max(V(t), 0)}{N(t)}\right]E[\delta(t-\tau)]\, dt\right].$$

The second expectation in the integrand is the default probability density function $p(t)$. The remainder of the integrand is the time zero value of the call option to enter into the tail of the swap at time t. If we denote this value by $C(0, t)$, then the CVA_V is:

$$(1 - R_V)\int_0^T C(0, t)p(t)dt.$$

Similarly, if $P(0, t)$ denotes the time zero value of the time t principal ($P(t)$), and it is deterministic, or otherwise independent of $V(t)$, then the CVA_P is:

$$-R_P \int_0^T N(0) E\left[\frac{P(t)}{N(t)} 1_{V(t)>0}\right] p(t)dt$$

$$= -R_P \int_0^T P(0,t)\,\Pr(V(t) > 0)\, p(t)dt,$$

where $\Pr(V(t) > 0)$ is the probability that $V(t)$ is positive. So the full CVA formula is:

$$(1 - R_V) \int_0^T C(0,t)p(t)dt - R_P \int_0^T P(0,t)\Pr\,(V(t) > 0)p(t)dt.$$

If the contract value is always positive, then the CVA becomes:

$$(1 - R_V) \int_0^T V(0,t)p(t)dt\ - R_P \int_0^T P(0,t)p(t)dt$$

$$= \int_0^T [(1 - R_V)V(0,t) - R_P P(0,t)]p(t)dt,$$

where $V(0,t)$ is the time zero value of the remainder of the contract at time t (the time t tail of the contract).

CVA Calculations for Bonds

If a risk free bond with a coupon of C pays f times per year at times t_i, with maturity t_n, the value of the bond is the discounted value of its cash flows:

$$\sum^{n} \frac{C}{f} D(t_i) + 100\, D(t_n),$$

where $D(t)$ is the risk free discount factor for time t (Stein, December 2007).

For bonds, recovery is on the principal ($R_V = 0$), the current value of the principal payment is the discounted value of the time at which it is paid, and the bond value is never negative. So,

$$P(0,t) = D(t)100,$$

and

$$V(0,t) = \sum_{t_i > t} \frac{C}{f} D(t_i) + 100\, D(t_n).$$

Assuming independence of rates and default, the counterparty valuation adjustment is:

$$\int_0^{t_n} [V(0,t) - R_P D(t) 100] p(t) dt.$$

Consider the contribution of one term from $V(0,t)$ to this integral. It is

$$\int_0^{t_i} \frac{C}{f} D(t_i) p(t) dt$$

$$= \frac{C}{f} D(t_i) \big(1 - S(t_i)\big),$$

where $S(t_i)$ is the survival probability for time t, (the probability of no default before time t, namely $1 - \int^t p(s)ds$). So, the CVA for the bond is:

$$\sum^n \frac{C}{f} D(t_i) \big(1 - S(t_i)\big) + 100\, D(t_n) \big(1 - S(t_n)\big) - R_P \int^{t_n} 100\, D(t) p(t) dt.$$

This is just the sum of the values of the discounted cash flows times the odds of losing them, minus the value of the recovery. Subtracting from the value of the risk free bond, we get the value of the risky bond:

$$\sum^n \frac{C}{f} D(t_i) S(t_i) + 100\, D(t_n) S(t_n) + \int^{t_n} 100\, R_P\, D(t) p(t) dt.$$

The value of the risky bond is the sum of the discounted cash flows times the odds of getting them plus the value of the recovery of the principal in the event of default. This is the standard CDS model applied to a bond.

To get a feel for the impact of the credit spreads on bond values, we can compute the credit adjusted par curve. For $t_i = i/f$, and for each n, solve for $C = C(t_n)$ such that

$$100 = \sum^n \frac{C}{f} D(t_i) S(t_i) + 100\, D(t_n) S(t_n) + \int^{t_n} 100\, R\, D(t) p(t) dt$$

Then $C(t)$ gives the credit adjusted implied par curve – the coupons that an issuer with this CDS spread curve would theoretically use to issue debt at par.

We do this calculation in the YASN function – Bloomberg's structured notes analysis function (See Figure 1).

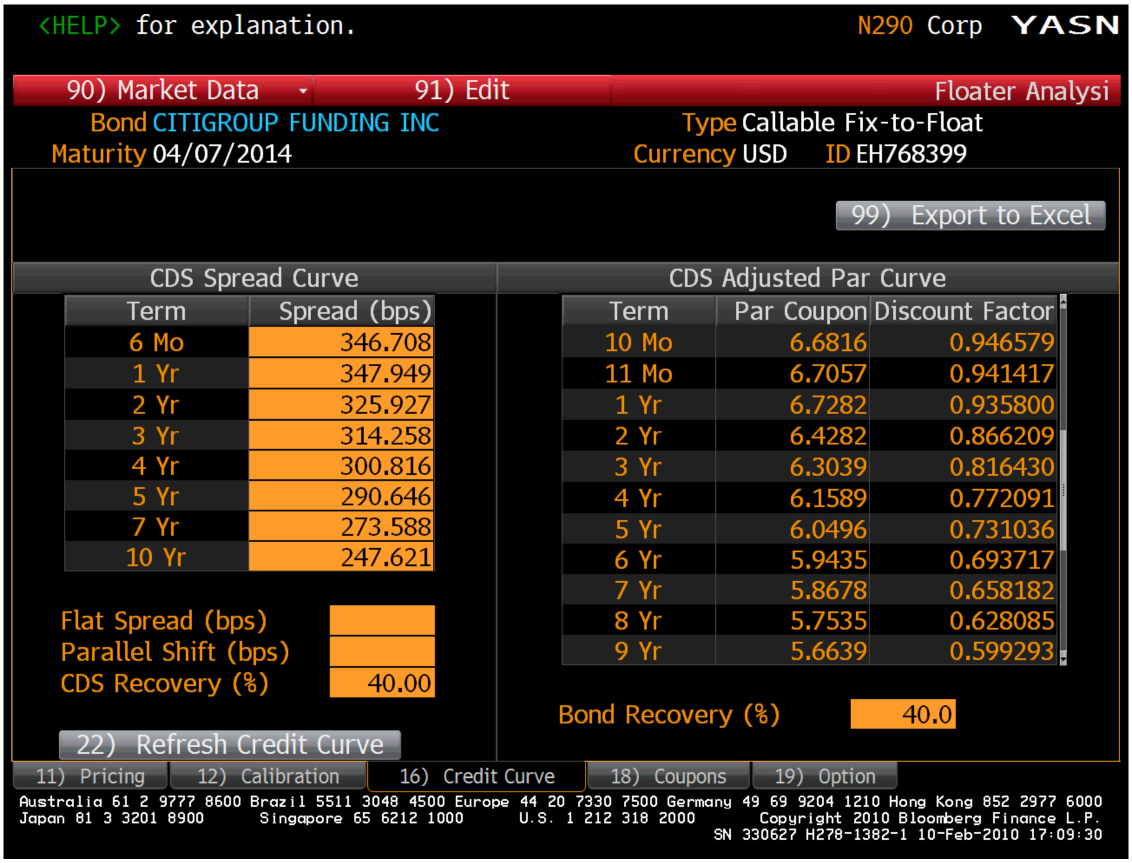


Figure 1. YASN, the structured notes analysis function, converts CDS curves to credit adjusted par curves for the purpose of doing OAS analysis on structured notes.

With a flat 100 bp CDS spread curve, and a flat 3% swap curve, the entire process roughly corresponds to adding the CDS spread to the swap par rates (aside from the impact of default between trade date and settlement date, which increases the par coupon at short maturities). When the CDS recovery rate differs from the bond recovery rate (or when CDS spreads are very large), the equivalent par curve can be significantly different from a shift of the risk-free curve by the credit spread, although it is still a shift. When the CDS and swap curves are not flat, this relationship no longer holds.

CDS Recovery Bond Recovery Term	0% 0%	40% 40%	80% 80%	40% 0%
1 Wk	4.22	4.22	4.24	5.08
1 Mo	4.00	4.01	4.02	4.71
1 Yr	4.02	4.04	4.11	4.72
2 Yr	4.03	4.04	4.09	4.73

3 Yr	4.03	4.04	4.09	4.73
5 Yr	4.03	4.04	4.08	4.72
7 Yr	4.03	4.04	4.08	4.72
10 Yr	4.03	4.04	4.08	4.72

Table 1. Credit adjusted par rates for a flat 100 bp CDS spread curve and a flat 3% swap curve as a function of the CDS and bond recovery rates.

Once the risky par curve is constructed, we can use it directly for valuing other bullet bonds by stripping it and applying the resultant discount factors to the bond cash flows. This gives a simple, straight forward way of incorporating CDS spreads into bond valuations. It is also an improvement over shifting the par curve in that it more accurately reflects the impact of the CDS spreads and also takes into account the shape of the credit spread curve. It also properly prices par bonds back to par.

In fact, if the bond recovery rate is zero, this method is exact. In this case, the risky discount factors are

$$D(t)S(t),$$

which makes the risky spot rate curve

$$\bar{R}(t) = -\frac{1}{t}\log\big(D(t)S(t)\big) = R(t) - \frac{1}{t}\log S(t),$$

where $R(t)$ is the risk free spot curve (in continuous compounding). In this case, the survival probabilities add a spread of $-\frac{1}{t}\log S(t)$ (the average hazard rate) to the risk free rate. This is roughly the CDS spread adjusted by the CDS recovery rate.

However, as bonds deviate from par and the recovery rate grows, this method will deviate from the CDS model based method. It can lead to 5 to 15 basis points of error with a 100 bp CDS spread when a bond is several hundred basis points away from par. However, given other general uncertainties (such as the value of the recovery rate itself, or the spread between the bond market and the CDS market), it is a reasonable approximation, and errors can be folded into the option adjusted spread (OAS) as an additional shift of the forward curve.

Once a bond has a more complicated structure (such as embedded calls or puts, or complicated floating coupons, such as range accruals), tone must do more than just discount cash flows. For complicated bonds, one can use the risk adjusted par curve with an interest rate model, as is done in the YASN function, where we calibrate the Hull-White short rate model with time dependent volatility to the swaptions that most closely hedge the bond. (See Figure 2) The model is then calibrated to the risky par curve by imposing a time dependent shift of the model's forward rates. We have found this approach to be a reasonable compromise between accuracy, complexity and tractability.

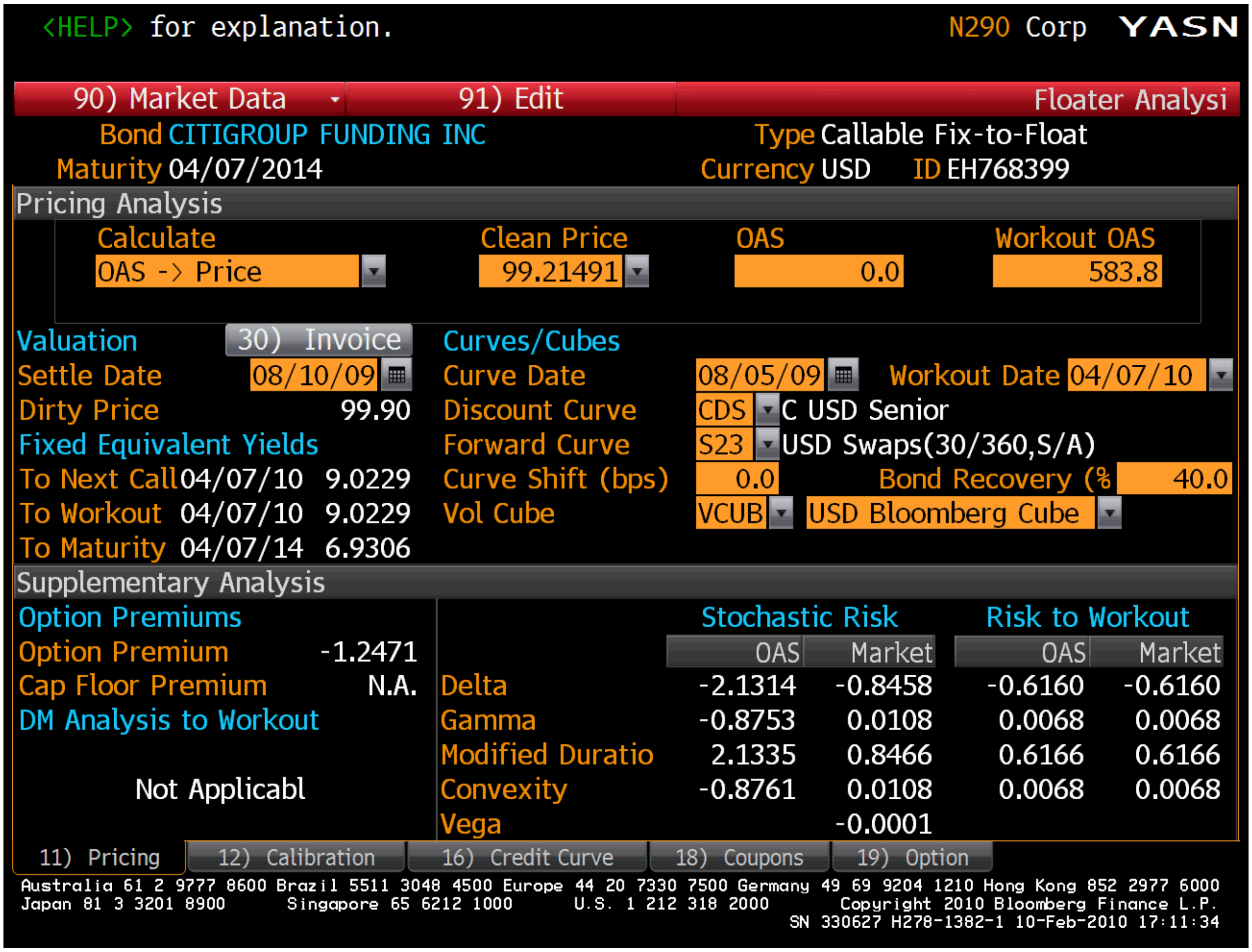


Figure 2. The YASN function applies volatility to the swap curve and is calibrated to quoted swaptions. The risk adjusted par curve is applied on top of this process to value risky cash flows.

CVA Calculations for Swaps

CVA calculations for swaps demand more finesse. The fundamental difference between the calculations for swaps and for bonds is that swaps can be assets or liabilities. When they are liabilities there is no counterparty default exposure. Taking this into account requires the introduction of swaptions.

For a swap, the principal recovery rate is zero, and the swap can be positive or negative in value. So, assuming independence of default and the contract price, the CVA is:

$$(1 - R_V) \int_0^T C(0,t) p(t) dt.$$

where $C(0,t)$ is the value of a call option maturing at time t to enter into the tail of the swap. Thus, we see that the CVA for a swap is sensitive to volatility.

We note that this formula holds more generally than just for interest rate swaps. This holds whenever default and the value of the security are independent, and recovery is a percentage of the risk free value of the security.

The value of the option on the tail, $C(0,t)$, differs from the usual value of a swaption with exercise time t in that in the swaption the first coupon is prorated, whereas in the option to enter into the tail, the loss is of the entire first coupon. The difference between the two payoffs is illustrated in Figure 3. The difference is further exacerbated by the fact that in a typical swap, the fixed and floating cash flows have different periodicities. As can be seen in Figure 4, since the first payment in a swaption is prorated, the forward swap values are much smoother than for the values of the tail of the swap in question.

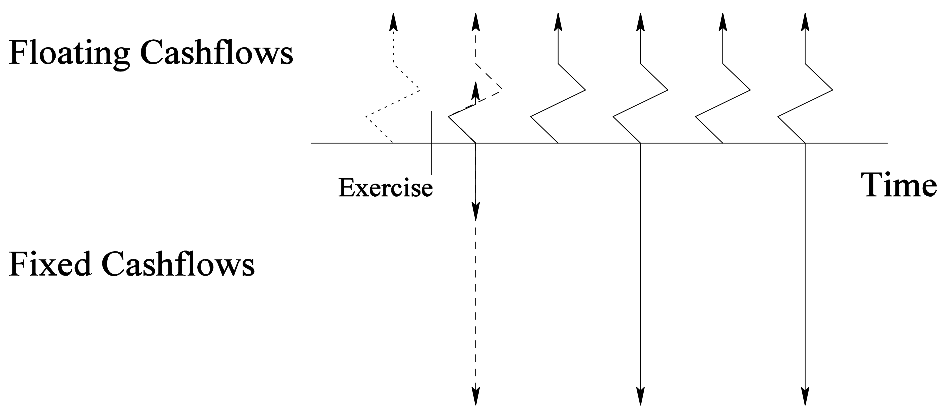


Figure 3. A swaption exercised between coupon payments would get the cash flows illustrated in bold, whereas upon default the full cash flows are lost.

The difference in forward values has an analogous impact on the value of the corresponding options, as illustrated in Figure 5. Again, while the swaption values are relatively smooth as a function of maturity, the option to enter into the tail of the swap jumps as cash flow dates are passed. One can also see that volatility has a huge impact on the overall exposure, with peak expected exposure changing from $5,957 to $14,418, and with the peak occurring earlier as well (see Figure 6).

To calculate the CVA for a swap, we approximate the above integral by a sum. For optimal accuracy, the calculation could be done on a daily basis, but this is extremely time consuming. Since default probabilities and option prices are relatively linear between cash flows, we can get good approximations if we a) use the midpoint of the interval for the option exercise, and b) take care to adjust for the difference between swaptions and options to enter into the tail of the swap.

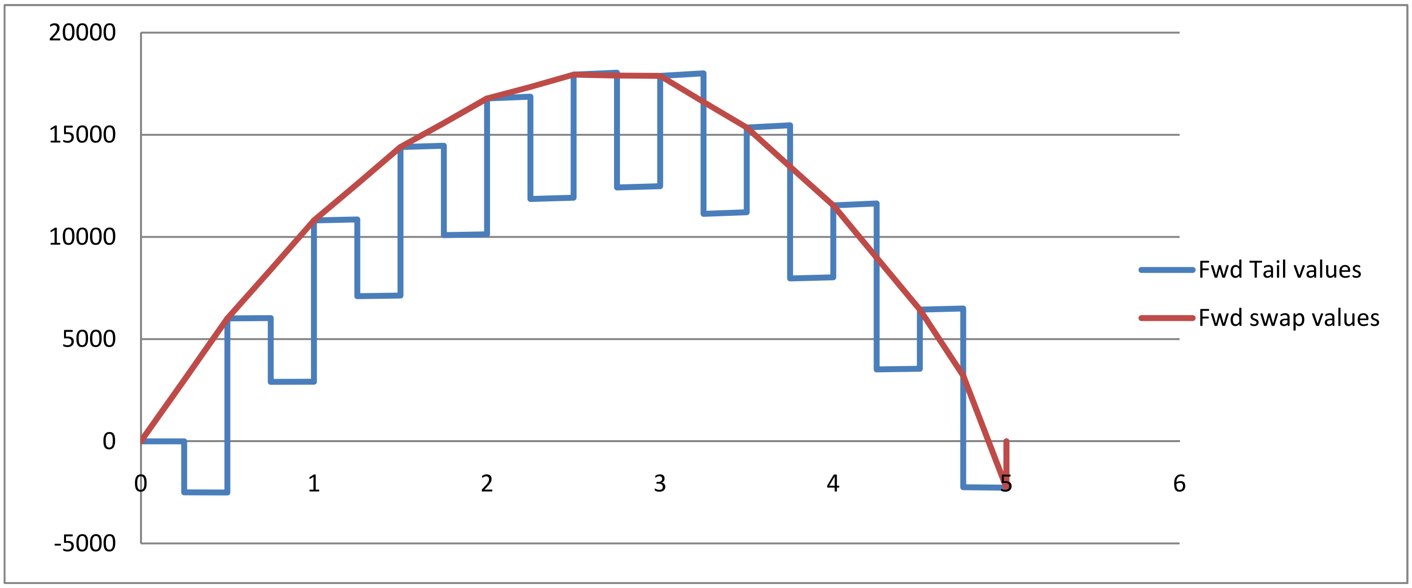


Figure 4. Comparison of forward swap values to forward tail values for a 5 year at the money payer swap.

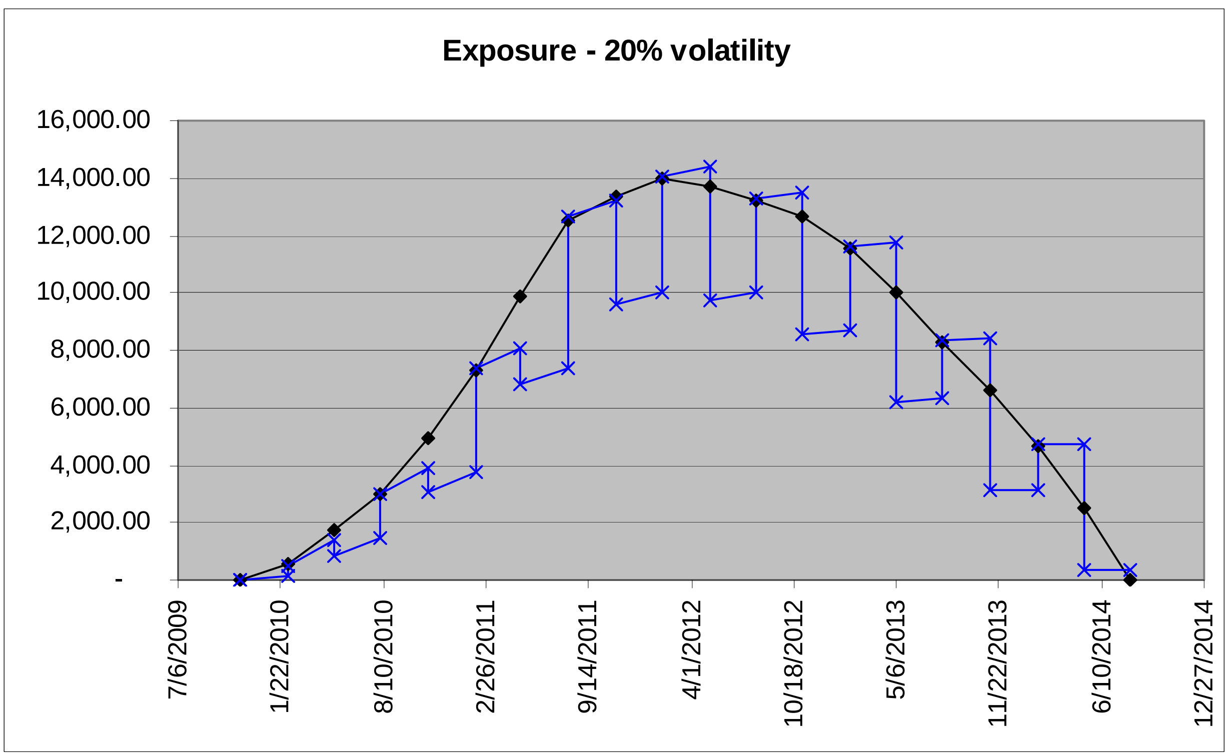


Figure 5. Swaption prices as a function of maturity are given by the blue line. The prices of options to enter into the tail of an existing payer swap are given by the black line. The underlying swap is a 5 year swap on a $1 million notional paying 4% fixed against 3 month US Libor. The options and swaptions are valued under a flat 20% volatility.

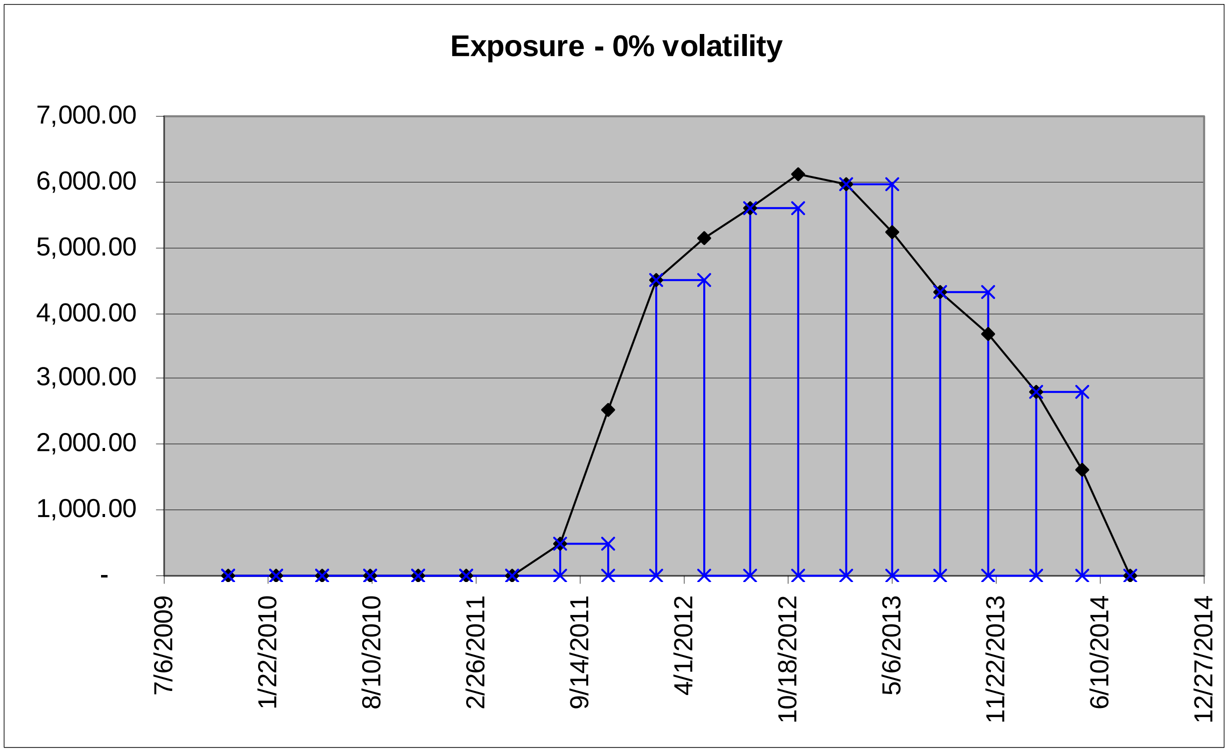


Figure 6. The same comparison as in Figure 5, but with 0% volatility.

The difference between the swaptions and the tail options can be accounted for by adjusting volatilities, strikes and forward rates. Consider the pay fixed swap with fixed rate F that the holder of a swaption would receive on exercise (at time t). Let the underlying floating (fixed) leg pay at times t_i (t_i'). Let $L(w,x,y)$ be the time w forward Libor rate setting at time x and paying at time y, $Z(x,y)$ be the time x price of a zero coupon bond paying at time y, and α_i (α_i') be the accrual fractions for the floating (fixed) payment periods from t_i to t_{i+1} (from t'_i to t'_{i+1}). Then the time t value of the swap is:

$$S(t) = \sum L(t, t_i, t_{i+1}) Z(t, t_{i+1}) \alpha_i - \sum F\, \alpha_i'\, Z(t, t_i').$$

Since $t \leq t_1$,

$$L(t, t_i, t_{i+1}) = \left(\frac{1}{\alpha_i''}\right)\left(\frac{Z(t, t_i)}{Z(t, t_{i+1})} - 1\right),$$

(with Libor accrual factor α_i''). If we assume $\alpha_i = \alpha_i''$, then $S(t)$ can be written in terms of Z as:

$$S(t) = Z(t, t_1) - Z(t, t_n) - \sum F\, \alpha_i' Z(t, t_i').$$

This gives us the standard expression for the time t value of a forward start swap (Stein, December 2007).

If we use $\overline{t_i}, \bar{\alpha}_i$, etc, to denote the same information for the option to enter into the tail of the existing swap, then all the values are the same, except that the first reset date ($\overline{t_1}$) is earlier than the valuation time (t), and the first accrual factors correspond to the full period rather than the partial period. The time t value of the first floating cash flow is:

$$\frac{Z(t,t_2)}{Z(\overline{t_1},t_2)} - Z(t,t_2),$$

So the value of the tail is:

$$\bar{S}(t) = \frac{Z(t,t_2)}{Z(\overline{t_1},t_2)} - Z(t,t_2) + \sum_2 L(t,t_i,t_{i+1})Z(t,t_{i+1})\alpha_i$$

$$- F\,\bar{\alpha}'_1 Z(t,t'_1) - \sum_2 F\,\alpha'_i Z(t,t'_i)$$

$$= \frac{Z(t,t_2)}{Z(\overline{t_1},t_2)} - Z(t,t_n) - F\,\bar{\alpha}'_1 Z(t,t'_1) - \sum_2 F\,\alpha'_i Z(t,t'_i).$$

The difference between the two is

$$\bar{S}(t) - S(t) = \frac{Z(t,t_2)}{Z(\overline{t_1},t_2)} - Z(t,t_1) - F\,(\bar{\alpha}'_1 - \alpha'_1)Z(t,t'_1).$$

This is the adjustment that needs to be made to convert the swaption payoff to the payoff of the option on the tail of the swap. If $D = \bar{S}(t) - S(t)$, then the payoff of the option to exercise into the tail of the swap is

$$\max(\bar{S}(t),0) = \max(S(t) + D,0) = \max(S(t),-D) + D.$$

This can be approximated by replacing D by its time zero value, and using it to approximate the value of the option on the tail by the swaption with an adjusted strike, forward rate, and volatility.

The impact of these adjustments is illustrated in Table 2. Cash flows are annual, yet annual sampling at the beginning or end of the period gives rise to substantial errors. Midpoint sampling is quite accurate even when just sampling once per year. The table also illustrates the impact of ignoring the difference between the options to enter into the tail of the swap and the swaptions.

sampling freq (in month)	CVA (anticipated)	CVA (postponed, using swaptions)	CVA (postponed, using tail options)	CVA (midpoint, using tail options)
2	$ 4,468.03	$ 4,529.87	$ 4,763.81	$ 4,615.92
4	$ 4,324.49	$ 4,445.17	$ 4,924.06	$ 4,613.98
6	$ 4,132.73	$ 4,315.81	$ 5,070.28	$ 4,651.73
12	$ 3,635.04	$ 3,953.62	$ 5,482.16	$ 4,656.87

Table 2. We compare computing the CVA using swaptions at the beginning of the default period (anticipated), at the midpoint, and at the end of the period (postponed). The underlying interest rate swap is a 5 year swap on $1 million notional, paying a 3.5% fixed coupon annually against 12 month U.S. LIBOR, with a 40% recovery rate and a flat 500 bp CDS spread. Calculations use market quotes of swap rates and swaption prices.

Example Calculation

Consider a 10 year ATM interest rate swap receiving fixed and paying US Libor 3 month on a $1 million notional. At a recovery rate of 40% and a flat CDS spread of 100 bp, the CVA is $2,174.68, as is illustrated by the Bloomberg CVA function (Figure 7).

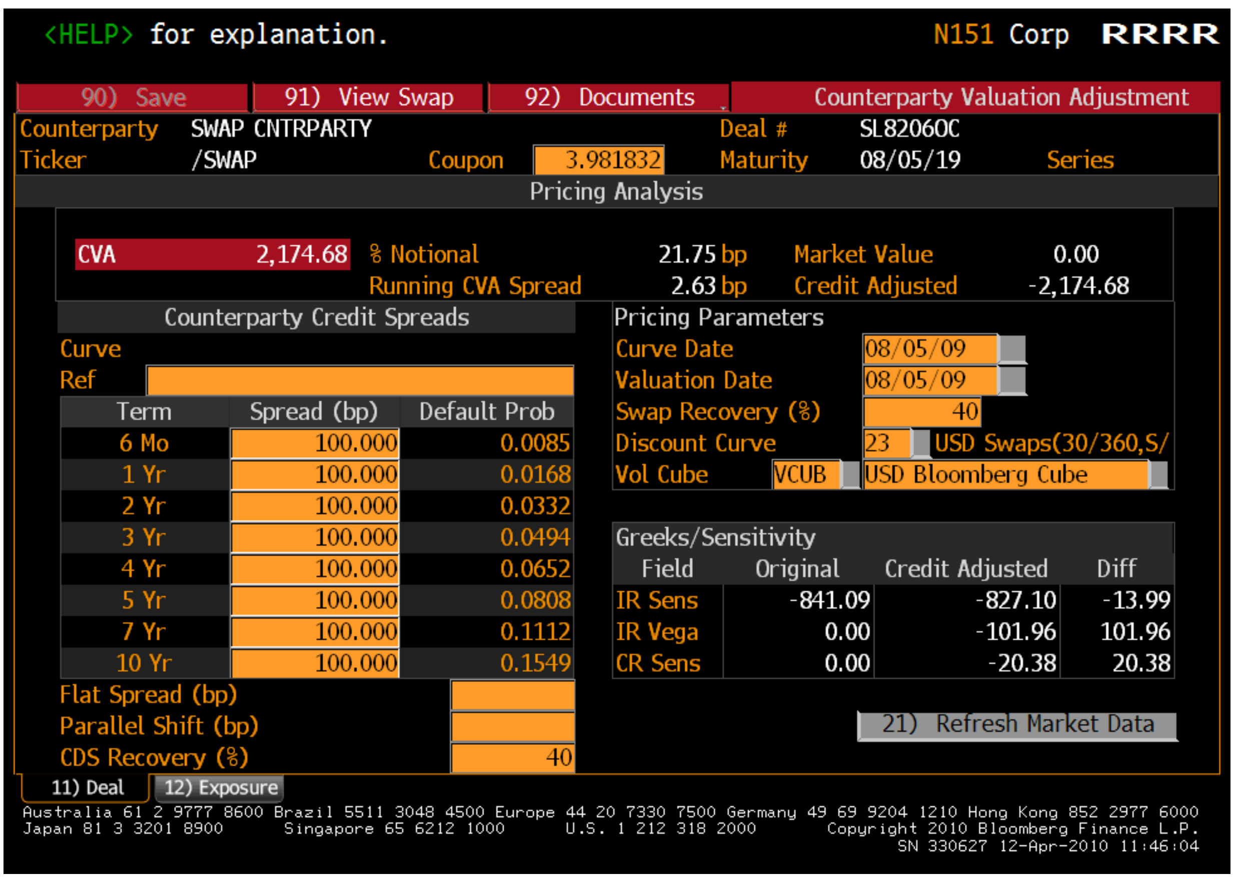
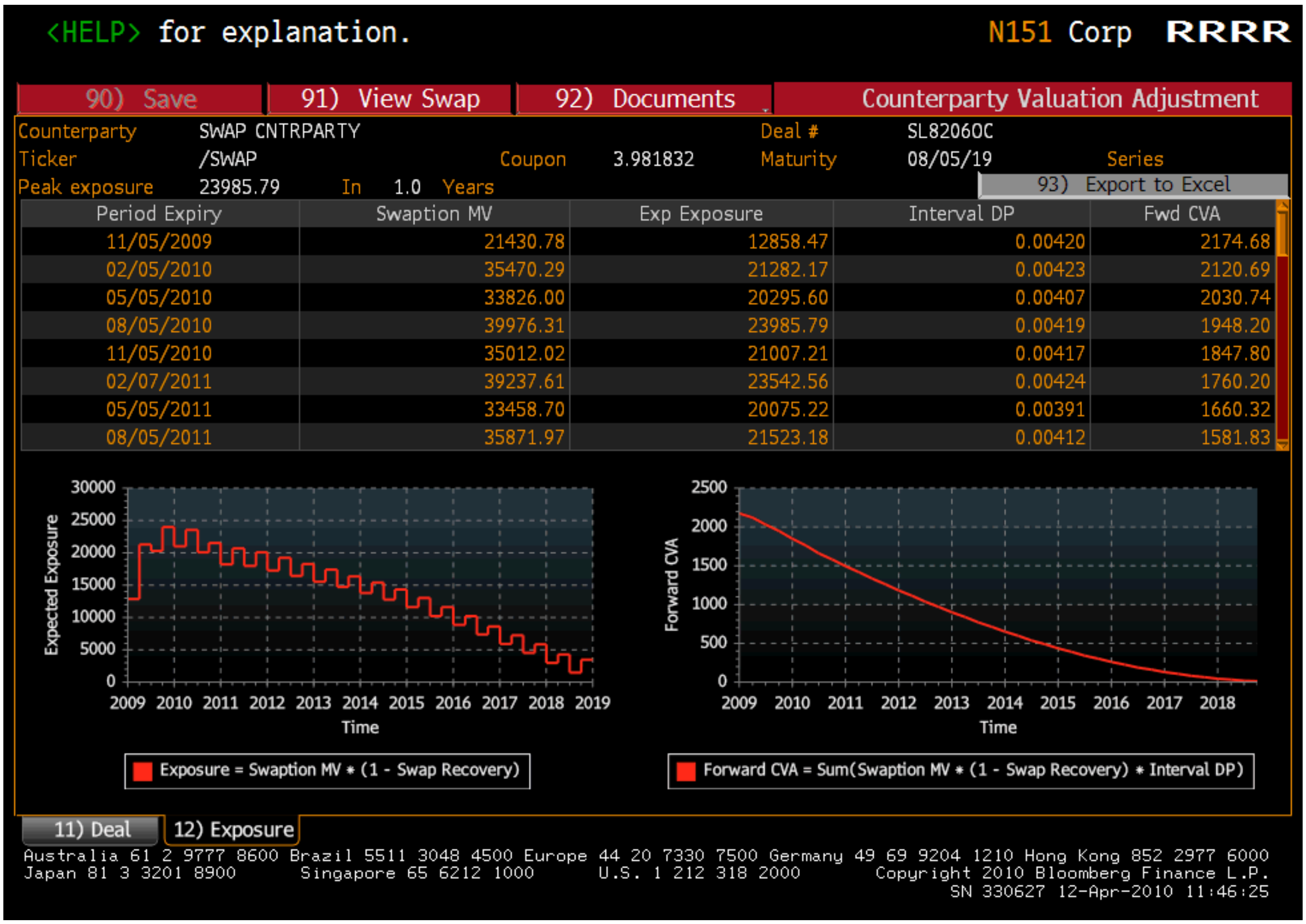


Figure 7.The CVA function computes counterparty valuation adjustments for OTC derivatives. Here we compute the CVA for a 10 year ATM receive fixed interest rate swap on $1 million notional with a 40% recovery rate and a flat 100 bp CDS spread. Calculations are using the swap curve and swaption volatilities from 8/5/09.

As can be expected, not only is the risky swap less valuable than the riskless swap, it also has credit and volatility exposure.

The details of the calculation are available on the second page (Figure 8). Here we see the interval default probabilities used with each swaption, the resultant present value of each potential loss, and the time t losses (assuming no default before that time).



Period Expiry	Swaption MV	Exp Exposure	Interval DP	Fwd CVA
11/05/2009	21430.78	12858.47	0.00420	2174.68
02/05/2010	35470.29	21282.17	0.00423	2120.69
05/05/2010	33826.00	20295.60	0.00407	2030.74
08/05/2010	39976.31	23985.79	0.00419	1948.20
11/05/2010	35012.02	21007.21	0.00417	1847.80
02/07/2011	39237.61	23542.56	0.00424	1760.20
05/05/2011	33458.70	20075.22	0.00391	1660.32
08/05/2011	35871.97	21523.18	0.00412	1581.83

Figure 8. The Exposure tab of the CVA function displays the swaption prices and default probabilities used in the CVA calculation.

It is interesting to note the difference in CVA values between payer and receiver swaps. In Figure 4, we show the forward values of a payer swap as a function of time. With an upward sloping yield curve, the swap is mostly of positive value. Contrast this with a receiver swap. In the same environment, a receiver swap's value will be the negative of this – the forward value will be negative over the life of the swap. Because of this, the payer swap spends much more time as an asset, and its CVA will be much higher than the equivalent receiver swap. For the swap in Table 3, the CVA goes up by a factor of 2 if the swap is flipped from payer to receiver. Table 3 shows the CVA for at the money payer and receiver swaps, along with the par coupon adjustments these CVAs would imply. More discussion of this subtlety can be found in (Bielecki, et al., 2002).

CDS Spread (bps)	Reciever swap CVA	Payer swap CVA	Receiver swap par coupon	Payer swap par coupon
10	$ 230.15	$ 484.59	3.985%	3.976%
20	$ 457.38	$ 962.68	3.987%	3.970%
50	$1,121.93	$ 2,358.79	3.995%	3.953%
100	$2,174.68	$ 4,563.37	4.008%	3.926%
200	$4,090.56	$ 8,548.83	4.032%	3.875%
300	$5,780.38	$12,028.01	4.054%	3.828%
400	$7,272.53	$15,063.43	4.074%	3.785%
500	$8,591.68	$17,709.67	4.092%	3.746%

Table 3. CVA and effective par coupon for payer and receiver swaps as a function of CDS spread. The underlying swap is a 10 year ATM swaps on $1 million notional with swap rate is 3.982%.

Hedging

Pricing is only as good as the hedges that can be used to lock it in. In the case of counterparty risk, the hedges can be problematic. Consider an interest rate swap. Assuming no correlation between default and interest rates, we know that the CVA is:

$$= (1 - R) \int_0^T C(0,t) p(t) dt$$
$$\cong (1 - R) \sum C(0,t_i) \bar{p}(t_i),$$

Where the t_i are chosen to be the midpoints of the intervals determined by the cash flows, and $\bar{p}(t_i)$ is the probability of defaulting in the i^{th} interval.

This suggests two ways of replicating the CVA. We can take positions $\bar{p}(t_i)$ in the corresponding swaptions, or we can take positions $C(0,t_i)$ in securities whose values are the interval default probabilities (if we can find them). Neither of these will actually replicate the CVA statically, because the positions themselves do not exhibit the same sensitivities to market conditions as the CVA does. But, but they can serve as a basis for hedging the CVA.

Consider hedging via the portfolio of swaptions. By holding the swaptions, this portfolio hedges against all changes of interest rates and volatilities. The portfolio itself is unaffected by credit spread changes. It would have to be rebalanced when credit spreads change.

Hedging the credit side is more complicated. When credit spreads increase, the CVA increases, so if we try to hedge with one credit default swap, we need to be long protection. Augmenting the above portfolio with the appropriate CDS position to neutralize the credit spread sensitivity will require being long CDS protection.

The main difficulty in using CDS in this fashion to hedge the CVA is that while we are hedged against shifts of the CDS spread curve, we are not necessarily hedged against the default

event[2]. To see this, consider having sold protection against default loss on an ATM swap. We would have been paid the CVA for this protection. To apply the above hedge, we would use the money to buy the above swaption portfolio and entered into this credit default swap. If default occurs immediately, because the swap is at the money, there would be no loss and we wouldn't have to pay anything. We could liquidate our swaption portfolio, recover the CVA value, and on top of that, because we are long protection from the CDS, receive additional income from the CDS position. Thus, an immediate default results in making money, so this position is not a perfect hedge – we end up over-hedged. Similarly, if the swap was heavily in the money and credit spreads were small, we could potentially be under-hedged by following this strategy.

Alternatively, we can take positions in CDS equal to the forward market values of the swap, minus the value of the corresponding swaption portfolio. This would properly hedge against the default event, but not necessarily hedge against changes in credit spreads. In either case, there is also the issue that the hedge bleeds – it incurs a loss over time due to the cost of the CDS protection. This loss would have to be factored into the hedge, which further complicates matters because it is also dependent on the default time.

In practice, since credit spreads change more frequently than default events occur, one typically hedges against credit spread changes with a portfolio of CDS positions. This hedge has to be managed in an attempt to make sure that one is sufficiently long protection before default becomes imminent, lest one be caught with insufficient coverage when the default event actually occurs. This can be managed to the extent that the default arrival is not Poisson (i.e. – there is some knowledge in the market about the default time beyond just the hazard rate).

Another hedging complication arises from the cross gamma (the sensitivity of the CVA to the second derivative with respect to credit spreads and interest rates). If interest rates or volatilities change, the swaption portfolio hedges the change. If credit spreads change, the CDS position hedges the change. The hedging problem that arises from the cross gamma is when both change at the same time. Because the CVA depends on the product of the default probabilities and the swaption values, the cross gamma is a function of the product of the first order derivatives, whereas the above hedge is not exposed to this product – it has roughly zero cross gamma. So when both change, the CVA move includes a cross gamma related move, while the hedge does not.

In all, more work can be done on practical, effective methods of hedging counterparty risk.

Wrong Way Risk and Recovery Risk

Assuming independence of the underlying position and the default event works only as long as the market complies with this assumption. Making this assumption can be a mistake when the counterparty has a close relationship with the underlying contract. When default risk increases

[2] This holds even if we expand the CDS spread to hedge against moves of all the points of the CDS spread curve by using a portfolio of CDS positions based on the sensitivity of the CVA value to moves of the individual curve points.

as the position value increases, this is known as "wrong way risk". One classic example is buying a put from a company on its own stock. The value of the put peaks when the company goes bankrupt and is unable to pay. Another example is entering into a receiver oil swap with an oil company. As oil prices drop, the company's profits drop, but the value of the swap increases. A more subtle example is in credit default swaps. Consider buying credit default swaps from one bank to insure against default of another bank. If the latter bank defaults, it could very well be in an environment where the former bank is unable to pay (Redon, April 2006).

Accounting for wrong way risk requires a model linking the value of the underlying contract with the default of the counterparty. For credit default swaps, this could be done by considering three separate events, each with their own hazard rate and Poisson arrival - the default of the first party, the default of the second party, and their joint default. Defaults are linked both through the joint default process and/or by driving the hazard rates by correlated processes (such as Brownian motions).

For contracts that do not involve defaults (like interest rate swaps), linking the counterparty default to the contract value is more subtle. One can, for example, make the hazard rate stochastic and driven by a Brownian motion which is correlated to the Brownian motions driving interest rates, but if the default event itself is a Poisson arrival (and hence uncorrelated to rates) this often doesn't yield as high a correlation as one would like. Some argue on this basis for structural models where default occurs when the firm value crosses a barrier (Lipton, et al., Summer 2009). None the less, even with a Poisson default arrival, correlation in this model still can have a sizable impact on the pricing (Brigo, et al., February, 2008).

The biggest difficulty in handling wrong way risk is in judging and hedging the correlation. While it is possible to model the correlation, how should one calibrate the model correlation to the market? Lacking derivatives that directly expose pricing of the correlation, one can only access the correlation indirectly, either through historical analysis or as a byproduct of the behavior of the model in question. In the former case, one is looking backward instead of forward, and is not necessarily operating under the pricing measure. In the latter case, one is assuming that certain characteristics of the market (like the shape of the volatility smile) are arising from the correlation. The lack of derivatives that directly expose the correlation makes hedging the correlation impact difficult as well.

Another weakness is related to recovery risk. Given that we will not know the recovery rate until some time after the default occurs, one should model the uncertainty of the recovery rate itself. Much work has been done on estimation of recovery rates and their incorporation into risk modeling (Altman, et al., December, 2003). Little work has been done in incorporating their uncertainty into CVA calculations, let alone calibrating and hedging it. One would expect difficulties similar to those in handling correlation, although some of the risk can be mitigated by credit default swaps, which are also exposed to recovery rate risk.

Accounting Considerations

Accounting for the counterparty risk in ones positions is required by accounting standards FASB 157 in the US and IAS 39 in Europe. For example, paragraph 5 of appendix B of FASB 157 states:

> B5. Risk-averse market participants generally seek compensation for bearing the uncertainty inherent in the cash flows of an asset or liability (risk premium). A fair value measurement should include a risk premium reflecting the amount market participants would demand because of the risk (uncertainty) in the cash flows. (Financial Accounting Standards Board, 2007)

While accounting rules are being updated (FASB and IAS are working jointly on refinements), the replacement for the above paragraph from FASB 157, namely Topic 820, Subtopic 10, Section 55, Paragraph 55, strengthens the position that credit risks need to be accounted for:

> 55-8. A fair value measurement should include a risk premium reflecting the amount market participants would demand because of the risk (uncertainty) in the cash flows. Otherwise, the measurement would not faithfully represent fair value. In some cases, determining the appropriate risk premium might be difficult. However, the degree of difficulty alone is not a sufficient basis on which to exclude a risk adjustment. (Financial Accounting Standards Board, 2009)

While accurate valuation of the embedded default risk is preferred, a number of alternative approaches have traditionally been accepted. The simplest approach is to follow the same method as is used for bonds, namely to shift the discount curve to account for the credit spread. When all the corresponding swaptions are in the money, this corresponds to the zero volatility version of the CVA. For swaps that are heavily in the money, this approach can give a rough idea of the CVA (as illustrated in Table 4). Here the volatility accounts for about 15% to 20% to the CVA.

CDS rate	CVA	Discount Shift
0	0	0
100	1,761	1,528
200	3,575	2,997
300	5,295	4,425
400	6,918	5,809

Table 4. CVAs computed with option volatilities compared to those computed by discount shifts for a heavily in the money swap – a 5% 5 year receiver swap on a $1 million notional, using market swap rates and market volatilities (the 5 year ATM swap rate is 3.16%).

If any of the relevant swaptions are out of the money, or the swaptions are close enough to at the money that the volatility plays a larger role, then this approximation can go seriously wrong. This can even happen for in the money swaps if the curve is sufficiently steep. The

errors are illustrated in Table 5, where we see that for an at the money swap, the discount curve shift underestimates the CVA by as much as 38%.

The worst case scenario for this methodology is an at the money swap in a flat interest rate environment. In this case, if the payment frequency of the two legs is equal, shifting the curve will have no impact on the swap value at all. The curve shift method will produce a zero CVA regardless of the default risk.

CDS rate	CVA	Discount Shift
0	0	0
100	1,185	735
200	2,287	1,432
300	3,311	2,092
400	4,262	2,719

Table 5. CVAs computed with option prices compared to those computed by discount shifts for a 5 year at the money receiver swap on a $1 million notional, using market swap rates and market volatilities (the 5 year ATM swap rate is 3.16%).

For an out of the money swap, this approximation cannot be used at all. It will cause an increase in the value of the swap, not an increase in the counterparty valuation adjustment. The same holds for pay fixed swaps in the typical positively sloped interest rate environment. For example, switching the above swap to a pay fixed swap would make all the discount shift values negative.

To compensate for the latter shortcoming, the discount method is modified. Positions that are of positive value are treated as assets and their value is reduced as above according to the counterparty's credit risk. Positions with negative values are treated as liabilities. Instead of depressing their value according to the counterparty's credit, their value is *increased* according to the *investor's* credit. This is by symmetry and based on the theory that the money is owed to the counterparty and only a fraction of it will be paid if the investor defaults. While treating positive and negative positions separately helps to salvage the discount shift approach, it also adds the problems that bilateral CVA causes. These will be discussed below.

Another approach that can be used is the current net exposure approach. This is a portfolio level method that can take netting agreements and collateralization into account. The sum of the values of the positions under a given netting agreement is the current net exposure. It is the amount that is subject to immediate default risk. If it is positive, then an immediate default of the counterparty would yield a loss for the investor. The loss could be insured by a position in a credit default swap against default of the counterparty with notional equal to the current net exposure. The maturity of the CDS contract would be chosen based on some notion of the average maturity or duration of the portfolio. The CVA would be the cost of the CDS contract's fixed leg (the leg paying the spread) which is often approximated by the credit spread for the CDS contract applied over the life of the contract. Collateralization can be taken into account

by reducing the current net exposure to the threshold level (the level beyond which the position must be collateralized).

The current net exposure approach can be analyzed in terms of the CDS hedge for the default risk of a portfolio. Considering that the forward values of the portfolio are varying, one could consider improving this method by using a portfolio of CDS contracts of varying maturities and notionals so that the net CDS notional at any given time matches the forward value of the portfolio. The set of CDS contracts with this property would then be the CDS hedge for the portfolio assuming zero interest rate volatility. The current net exposure approach is a rough approximation of this with just one CDS contract.

The advantages of the current net exposure approach are that it is very easy to implement and it easily takes netting agreements into account. On the negative side, being a zero volatility approach, it suffers from similar problems to the discount shift approach in that the valuations neglect the value coming from interest rate volatility, which can be substantial.

A practical problem with the method is that it tends to be unstable. The CDS notional fluctuates with market value of portfolio and so does the computed CVA. It is not dampened by the volatility as would the accurate calculation of the CVA itself. Consider, for example, the CVA calculations in Figure 7. The sensitivities show that a one basis point move of interest rates moves the market value by $827, but moves the CVA by only $14.

Another modification often found in accounting practices is to use what is known as the bilateral CVA instead of the above calculation of the embedded counterparty default risk (known as unilateral CVA). Instead of computing the value of the contract subject to default of the counterparty, the investor would compute the value subject to default of either the counterparty or himself. This complicates the calculation, but is often approximated as the difference between the investor's CVA to the counterparty and the counterparty's CVA to the investor. This approximation can have significant error depending on the relationships between when the contract is positive, when it is negative and who defaults first.

Bilateral CVA is often employed in accounting. It is also looked upon favorably by investors because it reduces the CVA charge. However, it has a number of undesirable properties. If the investor's credit is worse than the counterparty's, it can actually increase the value of the derivative. Similarly, a drop in credit of the investor could potentially cause an improvement in the balance sheet. Some, however, argue that this is a feature of mark to market accounting and not a drawback, noting that bonds issued by a firm will impact the balance sheet in a similar fashion. See (Gregory, February 2009) or (Brigo, et al., 2009) for further discussions. The latter also includes a detailed example of how CVAs are impacted by changes in credit spreads and correlations.

Another problem with bilateral CVA is that the price of a derivative should be associated with a hedge. But there is no realistic hedge an investor can establish for his own default risk. This renders the bilateral CVA less than a realistic market price.

Because of these issues, accounting boards have been lobbied to reject bilateral CVA as an acceptable approach. We can hope that these issues will be addressed as IASB, FASB and other accounting standards boards work together on global convergence of accounting standards.

Conclusions

The financial crisis that began in 2007 has highlighted the importance of assessing counterparty credit risk. Counterparty credit risk can be quantified by the credit valuation adjustment (CVA). The CVA for a bond and other securities that are long only can be calculated by using curve shifts. However, for securities that combine long and short positions (such as interest rate swaps), discount curve shifts are of limited utility. In this case, calculations must be done taking volatility into account, and essentially consist of combining appropriately adjusted swaption prices with default rates (as can be derived from CDS spreads). We have:

- Outlined the general CVA calculation
- Detailed the application to bonds and interest rate swaps
- Presented hedging methodologies
- Discussed the impact of the underlying model assumptions
- Analyzed the relationship between the CVA and some of the commonly accepted approaches to accounting for credit risk on balance sheets.

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